INVESTIGATION ON ENERGY LEVELS OF BOTTOMONIUM

Yin Min Thant[1], Aye Thandar Htay[2]

Abstract

Energy levels of the bottomonium are determined by solving Schrödinger equation with Numerov method. In this paper, we solved the non-relativistic Schrödinger equation theoretically with screened potential that includes spin dependent terms in the potential. It was used to predict the bottomonium η_b's (singlet S and D states), Υ's (triplet S and D states), h_b's (singlet P and F states) and χ's (triplet P and F states).

Keywords: bottomonium states, Numerov's method, screened potential

Introduction

Bottomonium is the bound state of a bottom quark (b) and its anti bottom quark ($\bar{b}$). The bottom quark was first described theoretically in 1973 by physicists Kobayashi and Maskawa and name "bottom" was introduced in 1975 by Harari [M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49** (1973) 652, H. Harari, Phys. Lett. **B 57** (1975) 265]. The mass of bottom quark is about 4.2 GeV/c^2. It is four times the mass of a proton and many orders of magnitude larger than common "light" quarks such as up, down and strange. If one attempts to separate a quark-antiquark pair, the energy of the gluon field becomes larger and larger until a new quark-antiquark pair can be created. As a result, one does not end up with two isolated quarks but with new quark-antiquark pairs instead. This absolute imprisonment of quarks is called quark confinement. So, it is impossible to observe a free quark in nature. An illustrative picture depicting the quark confinement is shown in Fig. 1.

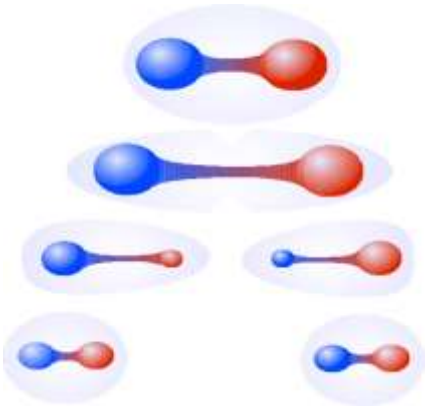

Figure 1 Quark confinement

Bottomonium States

The total spin of quark-antiquark bound state system have either spin singlet 'S = 0' or spin triplet 'S = 1'. The parity and charge-conjugation eigenvalues can be written as $P = (-1)^{L+1}$ and $C = (-1)^{L+S}$ respectively. In this equation, 'L' represents the orbital angular momentum. The notation S, P, D and F for states corresponds to L = 0, 1, 2 and 3. The states of the quark-antiquark bound state system can be represented by $n^{2S+1}L_J$; J and n stand for total angular momentum of quark-antiquark bound state system and the principle quantum number n = 1, 2, 3 and so on. As an example, the full notation for $\Upsilon(1S)$ is 1^3S_1.

[1] Department of Physics, University of Mandalay, Mandalay, Myanmar
[2] Department of Physics, Mandalar University, Mandalay, Myanmar

There are experimental observations of bottomonium states with different orbital angular momenta which are 'upsilon (ϒ)' for triplet S-states; ϒ(1S), ϒ(2S), ϒ(3S), ϒ(4S), ϒ(5S) and ϒ(6S), 'eta (η_b)' for singlet S-states; η_b (1S), η_b (2S), η_b (3S), η_b (4S), η_b (5S) and η_b (6S), 'chi (χ)' for triplet P-states; χ_b(1P), χ_b (2P), and χ_b (3P), h_b for singlet P-states; h_b (1P), h_b (2P) and h_b (3P), upsilon ϒ (1D) for triplet D-state and η_{b2} for singlet D-states respectively. In other words, 'upsilon and eta' are defined as the principal quantum numbers (odd) for triplet and singlet states and then 'chi and h_b' are assigned for even quantum numbers.

Potential Models for Interaction between Bottom and Antibottom

There are various non relativistic potential models to describe the bottomonium spectrum. Also potential models have been successful in describing the spectra below the open-flavor thresholds for both charmonia and bottomonia. However, it is well known that these potential models, which incorporates a Coulomb term at short distances and a linear confining potential at large distance. It is useful to improve the potential model itself to incorporate the screening effect and compare the model predictions with the experimental data as a phenomenological way to investigate the screening effects on heavy quarkonium spectrum [E. Eichen *et al*., Phys. Rev. **D17** (1978) 3090, E. Eichen *et al*., Phys. Rev. **D21** (1980) 203 and S. Godfrey *et al*., Phys. Rev. **D** 32 (1985) 189]. In calculation of bottomonium states, a non-relativistic potential model will use with the screening effect. The potential is described as

$$V(r) = V_v(r) + V_s(r) \tag{1}$$

where,

$$V_v(r) = -\frac{4}{3}\frac{\alpha_c}{r}, \tag{2}$$

$$V_s(r) = \frac{b(1 - e^{-\mu r})}{\mu}. \tag{3}$$

Here, $V_v(r)$ represents the vector-like one-gluon exchange potential, α_C is the coefficient of the Coulomb potential and μ is the screening factor which makes the long- range scalar potential of $V_S(r)$ behave like "br" when $r \ll \frac{1}{\mu}$, and become a constant b/μ when$r \gg \frac{1}{\mu}$. The main effect of the screened potential on the spectrum is that the masses of the higher excited states are lowered. Vector potential $V_V(r)$, scalar potential $V_S(r)$, and vector potential $V_V(r)$ plus scalar potential $V_S(r)$ are shown in Fig. 2, 3 and 4 respectively. The screening effect is very important to describe the higher excited states. In calculation, spin-dependent potentials include as follows:

(i) the spin-spin contact hyperfine potential

$$H_{SS} = \frac{32\pi\alpha_c}{9m_b^2}\left(\frac{\sigma}{\sqrt{\pi}}\right)^3 e^{-\sigma^2 r^2}\vec{S}_b.\vec{S}_{\bar{b}} \tag{4}$$

where $\vec{S}_b$and $\vec{S}_{\bar{b}}$ are spin matrices acting on the spins of the quark and antiquark.

In the $|^{2S+1}L_J\rangle$ basis, the matrix element for the spin-spin operator $\vec{S}_b.\vec{S}_{\bar{b}}$is

$$\langle\vec{S}_b.\vec{S}_{\bar{b}}\rangle = \frac{1}{2}S(S+1) - \frac{3}{4}. \tag{5}$$

(ii) the spin-orbit term,

$$H_{SL} = \frac{1}{2m_b^2 r}\left(3\frac{dV_v}{dr} - \frac{dV_S}{dr}\right)\vec{L}.\vec{S} \quad (6)$$

$$= \frac{1}{2m_b^2 r}\left(4\frac{\alpha_c}{r^2} - be^{-\mu r}\right)\vec{L}.\vec{S} \quad (7)$$

The matrix element of the spin-orbit operator is

$$\langle \vec{L}.\vec{S} \rangle = \frac{1}{2}[J(J+1) - L(L+1) - S(S+1), \quad (8)$$

where, L is the orbital angular momentum quantum number, S is the spin quantum number and J = L ± S is the total angular momentum quantum number. [W. Deng, *etal*., Phys. Rev. **D95** (2017) 074002]

(iii) the tensor term,

$$H_T = \frac{1}{12m_b^2}\left(\frac{1}{r}\frac{dV_v}{dr} - \frac{d^2V_v}{dr^2}\right)\vec{S}_T. \quad (9)$$

The element of the tensor operator S_T is

$$S_T = 4\langle \vec{S}^2\vec{L}^2 - \frac{3}{2}\vec{L}.\vec{S} - 3(\vec{L}.\vec{S})^2 \rangle. \quad (10)$$

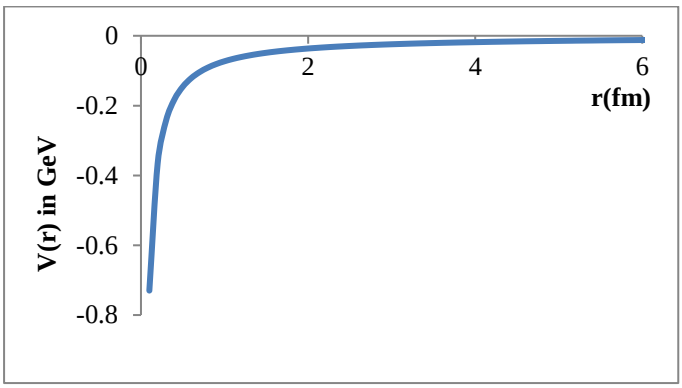

Figure 2 Vector potential V_V (r) in GeV with the distance r in fm

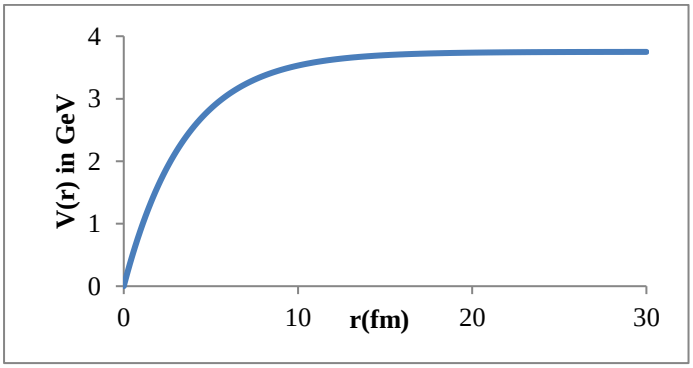

Figure 3 Scalar potential V_S (r) in GeV with the distance r in fm

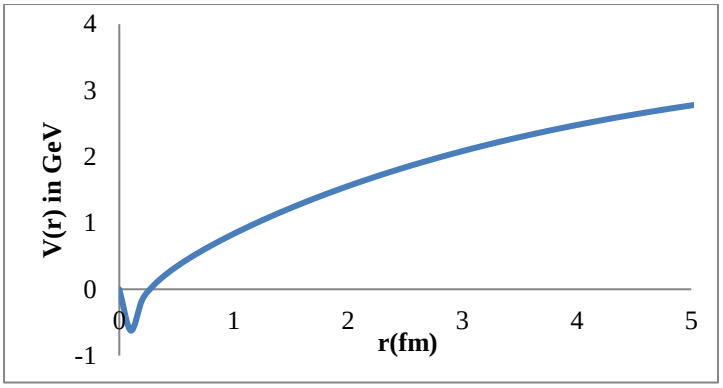

Figure 4 Vector potential $V_V(r)$ plus scalar potential $V_S(r)$

Energy Levels of Bottomonium

Energy levels of the bottomonium are determined by solving Schrödinger equation with Numerov method. To obtain energy (masses) and wave functions of the bottomonium states, this radial equation need to solve.

$$\frac{d^2u(r)}{dr^2}+\frac{2\mu}{\hbar^2}\left[E-V_{b\bar{b}}(r)-\frac{\hbar^2\ell(\ell+1)}{2\mu r^2}\right]u(r)=0 \qquad (11)$$

where,

$$\mu=\frac{m_b m_{\bar{b}}}{m_b+m_{\bar{b}}} \qquad (12)$$

μ is the reduced mass of the system and E is the binding energy of the system. Then, the mass of a $b\bar{b}$ state is obtained by $M_{b\bar{b}} = 2m_b + E$. In this calculation, screened potential model with spin-spin term, spin-orbit term and tensor term described as the following equation,

$$V_{b\bar{b}}(r)=V(r)+H_{SS}+H_{SL}+H_T. \qquad (13)$$

Therefore, our potential form is

$$V_{b\bar{b}}(r)=-\frac{4}{3}\frac{\alpha_c}{r}+\frac{b(1-e^{-\mu r})}{\mu}+\frac{32\pi\alpha_c}{9m_b^2}\left(\frac{\sigma}{\sqrt{\pi}}\right)^3 e^{-\sigma^2r^2}\vec{S}_b.\vec{S}_{\bar{b}}$$
$$+\frac{1}{2m_b^2 r}\left(4\frac{\alpha_c}{r^2}-be^{-\mu r}\right)\vec{L}.\vec{S}+\frac{1}{12m_b^2}\left(\frac{1}{r}\frac{dV_v}{dr}-\frac{d^2V_v}{dr^2}\right)\vec{S}_T. \qquad (14)$$

We calculated energy eigenvalues and the corresponding eigenfunctions by using numerical Numerov method in FORTRAN programming.

Results and Discussion

Energy Search for Bottomonium States

The Schrödinger equation has been solved with screen potential model including spin-orbit term and tensor term to obtain the energy levels of all bottomonium states. The angular momentum states S, P, D and F (L = 0, 1, 2 and 3) have been calculated.

Orbital Angular Momentum S (L = 0) States

Orbital angular momentum L = 0 states with spin singlet and triplet states are calculated by solving the Schrödinger equation with Numerov (Difference) numerical method. Our calculated masses of S-states are in good agreement with those found in literatures. The corresponding reduced radial wavefunction of bottomonium singlet and triplet states (5S, 6S) are expressed in Fig. 5 and 6. The comparison between the experimental data and our calculated masses of bottomonium for S states is shown in Table (1).

Higher Orbital Angular Momentum States (L =1, 2 and 3)

In this section, we explain the calculation of higher angular momentum states P, D and F which are L = 1, 2 and 3 by using the same procedure as that of S state calculation. We calculated the energy eigenvalues and eigenfunctions of higher orbital angular momentum states for principle quantum numbers n = 1, 2 and 3.

For these states the spin-orbit term is to be included and the term $1/r^3$ appears in the potential. Accordingly, when $r \to 0$ the wavefunction near the origin does not obey the relation $u(r \to 0) \propto r^{L+1}$. In order to overcome this problem, it is assumed that in a small range $r \in (0, r_c)$, the $V_{b\bar{b}}(r) \propto 1 / (r_c)^3$, which is a finite constant where r_c is a cut off distance. For the model parameters, we take $\alpha_C = 0.37$, $b = 0.210$ GeV^2, $\mu = 0.060$ GeV, $m_b = 4.760$ GeV and $\sigma = 3.10$ GeV. In our calculation, the cutoff distance $r_c = 0.06$ fm is adopted. With this cutoff distance r_c, the energy levels of the higher orbital states (L = 1, 2, 3) are in good agreement with the experimental data and the predicted masses of bottomonium [W. Deng, *et al*., Phys. Rev. **D95** (2017) 074002]. The comparison results are listed in Table (2) and (3). The reduced radial wavefunction of some bottomonium states are shown in Fig. 7-11. The experimental data and our calculated masses of bottomonium are shown in Fig. 13.

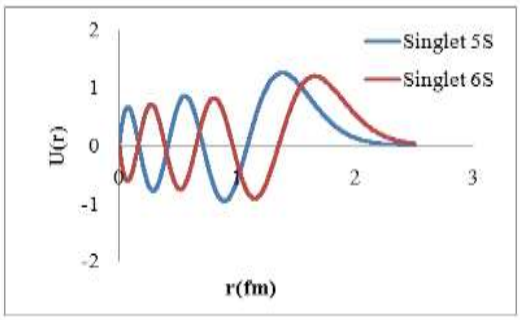

Figure 5 Reduced radial wavefunciton of bottomonium for singlet 5S and 6S state

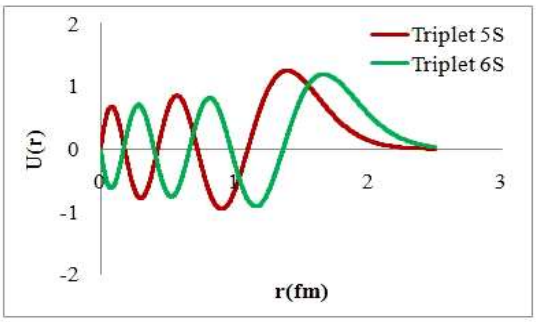

Figure 6 Reduced radial wavefunciton of bottomonium for triplet 5S and 6S state

Table 1 Comparison between the experimental data [PDG], predicted values [W. Deng, *etal*.,] and our calculated masses of bottomonium for S states

$n^{2S+1}L_J$	State	J^{PC}	Bottomonium masses (MeV)		
			[PDG]	Predicted Values	Our Calculated Results
1^3S_1	$\Upsilon(1S)$	1^{--}	9460	9460	9465.52
1^1S_0	$\eta_b(1S)$	0^{-+}	9398	9390	9393.59
2^3S_1	$\Upsilon(2S)$	1^{--}	10023	10015	10025.28
2^1S_0	$\eta_b(2S)$	0^{-+}	9999	9990	9999.46
3^3S_1	$\Upsilon(3S)$	1^{--}	10355	10343	10354.65
3^1S_0	$\eta_b(3S)$	0^{-+}		10326	10337.21
4^3S_1	$\Upsilon(4S)$	1^{--}	10579	10577	10609.26
4^1S_0	$\eta_b(4S)$	0^{-+}		10584	10595.65
5^3S_1	$\Upsilon(5S)$	1^{--}	10865	10811	10822.46
5^1S_0	$\eta_b(5S)$	0^{-+}		10800	10811.18
6^3S_1	$\Upsilon(6S)$	1^{--}	11020	10997	11007.88
6^1S_0	$\eta_b(6S)$	0^{-+}		10988	10998.20

Table 2 Comparison between the experimental data [PDG], predicted values [W. Deng, *etal*.,] and our calculated masses of bottomonium for P states

$n^{2S+1}L_J$	State	J^{PC}	Bottomonium masses (MeV)		
			[PDG]	Predicted Values	Our Calculated Results
1^3P_2	$\chi_{b2}(1P)$	2^{++}	9912	9921	9934.50
1^3P_1	$\chi_{b1}(1P)$	1^{++}	9893	9903	9903.95
1^3P_0	$\chi_{b0}(1P)$	0^{++}	9859	9864	9883.80
1^1P_1	$h_b(1P)$	1^{+-}	9899	9909	9918.64
2^3P_2	$\chi_{b2}(2P)$	2^{++}	10269	10264	10277.84
2^3P_1	$\chi_{b1}(2P)$	1^{++}	10255	10249	10254.65
2^3P_0	$\chi_{b0}(2P)$	0^{++}	10233	10220	10239.54
2^1P_1	$h_b(2P)$	1^{+-}	10260	10254	10265.23
3^3P_2	$\chi_{b2}(3P)$	2^{++}		10528	10541.79
3^3P_1	$\chi_{b1}(3P)$	1^{++}	10516	10515	10522.23
3^3P_0	$\chi_{b0}(3P)$	0^{++}		10490	10509.60
3^1P_1	$h_b(3P)$	1^{+-}		10519	10530.86

Table 3 Comparison between the experimental data [PDG], predicted values [W. Deng, *etal*.,] and calculated masses of bottomonium for D and F states

$n^{2S+1}L_J$	State	J^{PC}	Bottomonium masses (MeV)		
			[PDG]	Predicted Values	Our Calculated Results
1^3D_3	$\Upsilon_3(1D)$	3^{--}		10157	10172.25
1^3D_2	$\Upsilon_2(1D)$	2^{--}	10164	10153	10161.68
1^3D_1	$\Upsilon_1(1D)$	1^{--}		10146	10153.99
1^1D_2	$\eta_{b2}(1D)$	2^{-+}		10153	10165.27
2^3D_3	$\Upsilon_3(2D)$	3^{--}		10436	10450.98
2^3D_2	$\Upsilon_2(2D)$	2^{--}		10432	10441.71
2^3D_1	$\Upsilon_1(2D)$	1^{--}		10425	10434.87
2^1D_2	$\eta_{b2}(2D)$	2^{-+}		10432	10444.84
1^1F_3	$h_{b3}(1F)$	3^{+-}		10339	10352.07
1^3F_4	$\chi_{b4}(1F)$	4^{++}		10340	10356.45
1^3F_3	$\chi_{b3}(1F)$	3^{++}		10340	10350.58
1^3F_2	$\chi_{b2}(1F)$	2^{++}		10338	10346.00

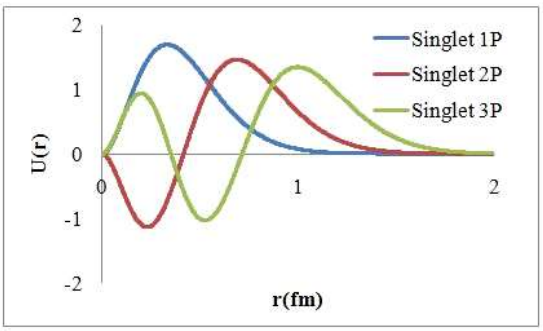

Figure 7 Reduced radial wavefunciton of bottomonium for singlet 1P, 2P and 3P states

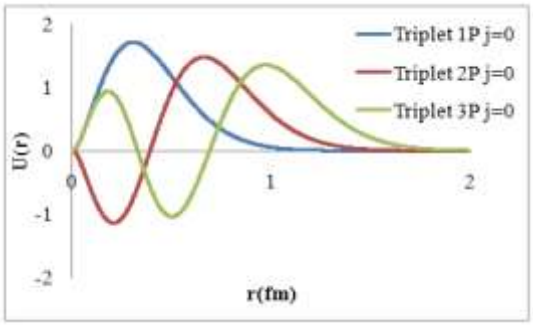

Figure 8 Reduced radial wavefunciton of bottomonium for triplet 1P, 2P and 3P(j = 0) states

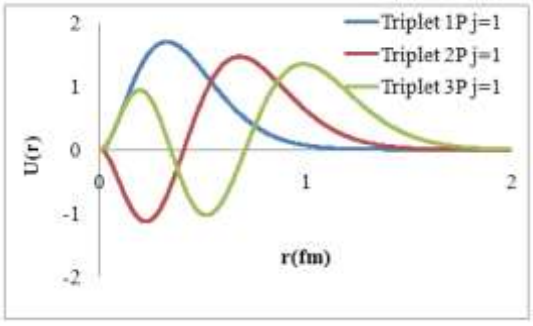

Figure 9 Reduced radial wavefunciton of bottomonium for singlet 1P, 2P and 3P (j = 1) states

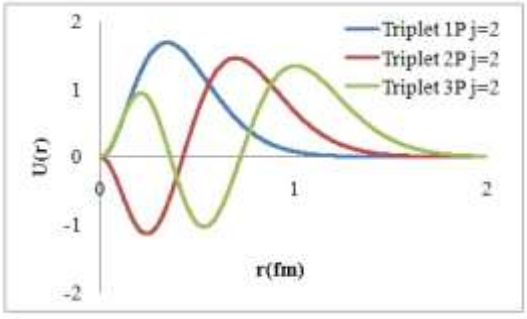

Figure 10 Reduced radial wavefunciton of bottomonium for triplet 1P, 2P and 3P(j = 2) states

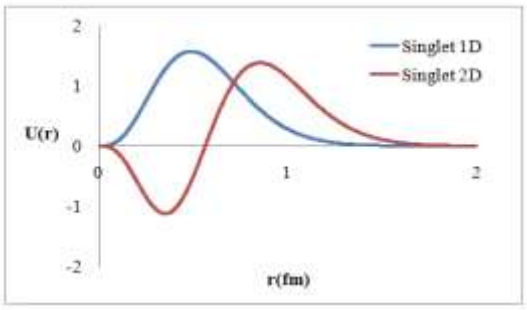

Figure 11 Reduced radial wavefunciton of bottomonium for singlet 1D and 2D states

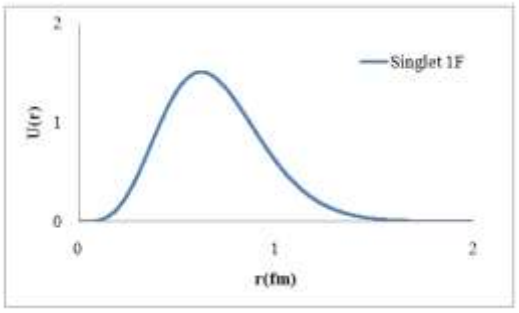

Figure 12 Reduced radial wavefunciton of bottomonium for singlet 1F state

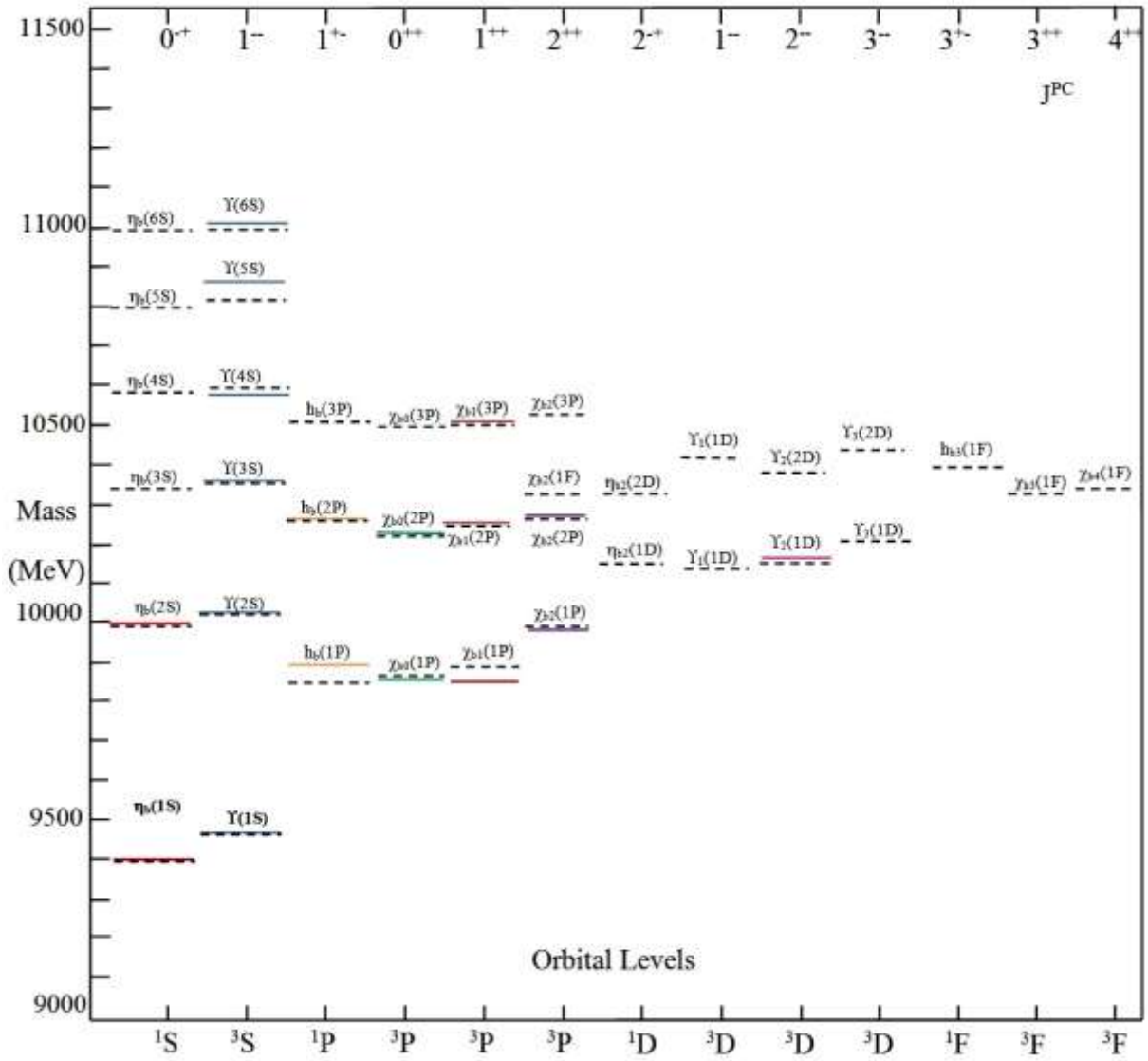

Figure 13 Masses of bottomonium where solid lines represent the experimental data and dashed lines the calculated ones

Conclusion

The masses of bottomonium states have been calculated in the non-relativistic screened potential model. Although the mass differences between the experimental data and our calculated values are 1 - 5 MeV for 1S, 2S and 3S states, 10 - 25 MeV for 1P state and 1 - 8 MeV for 2P, 3P and 1D states, our results for the bottomonium spectrum are in good agreement with the experimental data and the predicted values [K. A. Olive *et al.*, Particle Data Group Collaboration, Review of Particle Physics, Chin. Phys. **C 38**, (2014) and W. Deng, *et al*., Phys. Rev. **D95** (2017) 074002]. Therefore, our calculated results would be reasonable for determining the energy levels of bottomonium.

Acknowledgements

We would like to express our deep gratitude to Dr Khin Swe Myint, Rector (Retired), University of Mandalay for her supervision and valuable suggestions to complete the whole work. We would like to thank Professor Dr Nyein Wint Lwin, Professor and Head, Department of Physics, University of Mandalay for her kind permission to carry out this research paper.

References

E. Eichen *et al*., Phys. Rev. **D17** (1978) 3090.

E. Eichen *et al*., Phys. Rev. **D21** (1980) 203.

H. Harari, Phys. Lett. **B 57** (1975) 265.

K. A. Olive *et al.*, [Particle Data Group Collaboration], Review of Particle Physics, Chin. Phys. **C 38**, (2014).

M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49** (1973) 652.

S. Godfrey *et al*., Phys. Rev. **D** 3**2** (1985) 189.

W. Deng, *et al*., Phys. Rev. **D95** (2017) 074002**.**